What do we do if we have an exponent of $\frac{3}{4}$? Look at the following example.

Solve for x: $x^{\frac{3}{4}} = 8$.

We want to eliminate the exponent to leave us with $x^1 = x$. Recall that when raising a power to a power, we multiply the exponents. What number when multiplied with $\frac{3}{4}$ will give 1? The answer is $\frac{4}{3}$, which is the reciprocal of $\frac{3}{4}$.

Let's try it.

Solve for x: $x^{\frac{3}{4}} = 8$.

$\left(x^{\frac{3}{4}}\right)^{\frac{4}{3}} = 8^{\frac{4}{3}}$ Raise both sides to the $\frac{4}{3}$ power.

Note: $\left(x^{\frac{3}{4}}\right)^{\frac{4}{3}} = x^{\frac{3}{4}\cdot\frac{4}{3}} = x^1 = x$

$x = 16$ Simplify.

Check the solution in the original equation.

$x^{\frac{3}{4}} = 8$

$16^{\frac{3}{4}} \stackrel{?}{=} 8$

$8 = 8$ ✓

To solve equations with fractional or negative exponents

1. Isolate the expression containing the exponent.
2. Raise both sides to the reciprocal of the power.
3. Solve the resulting equation.
4. Check your answer in the original equation.

MODEL PROBLEM

Solve for y.

a. $3y^{\frac{3}{2}} = 192$

b. $(y + 3)^{\frac{5}{2}} = 32$

c. $2(3y - 4)^{\frac{3}{5}} - 4 = 50$

d. $\left(2y - \frac{5}{16}\right)^{-\frac{3}{4}} - 1 = 7$

e. $(y - 6)^{-\frac{1}{2}} + 5 = 3$

SOLUTION

a. $3y^{\frac{3}{2}} = 192$

$y^{\frac{3}{2}} = 64$ Isolate the y-value.

$\left(y^{\frac{3}{2}}\right)^{\frac{2}{3}} = 64^{\frac{2}{3}}$ Raise both sides to the $\frac{2}{3}$ power.

$y = 16$ Simplify.

Practice

In 1–16, solve and check. All variables represent positive numbers.

1. $x^{\frac{2}{3}} = 25$
2. $y^{\frac{3}{4}} = 125$
3. $z^{-\frac{5}{3}} = 243$
4. $2a^{-\frac{1}{4}} = 12$
5. $a^{\frac{3}{5}} - 2 = 25$
6. $2b^{-\frac{1}{3}} + 5 = 15$
7. $3r^{-\frac{3}{4}} = 81$
8. $-4s^{-\frac{3}{5}} - 1 = 31$
9. $(g - 1)^{\frac{1}{2}} = 5$
10. $(w + 1)^{-\frac{1}{3}} = 2$
11. $(r - 4)^{-\frac{1}{2}} - 1 = 5$
12. $2(v - 1)^{\frac{4}{3}} = 32$
13. $(3x - 1)^{\frac{3}{5}} = 125$
14. $(5z - 2)^{\frac{5}{3}} - 1 = 31$
15. $3(2m + 3)^{\frac{2}{3}} + 2 = 77$
16. $2(3y + 2)^{-\frac{5}{2}} = \frac{1}{16}$

In 17–20, select the numeral preceding the expression that best completes the statement or answers the question.

17. Which of the following equations does not have a solution in the set of real numbers?
    (1) $y^{\frac{2}{3}} = -4$ (3) $y^{-\frac{2}{3}} -8$
    (2) $y^{\frac{1}{3}} = -8$ (4) $y^{-\frac{1}{2}} = 5$
18. A root of $(z - 2)^{-\frac{3}{4}} = 8$ is
    (1) -2 (3) $\frac{31}{16}$
    (2) $-\frac{31}{16}$ (4) $\frac{33}{16}$
19. To solve the equation $\sqrt{(x + 1)^3} = 4$, we can rewrite the equation as
    (1) $\frac{2}{3}(x + 1) = 4$ (3) $(x + 1)^{\frac{2}{3}} = 4$
    (2) $\frac{3}{2}(x + 1) = 4$ (4) $(x + 1)^{\frac{3}{2}} = 4$
20. The solution of the equation $y^{\frac{2}{3}} = 8$ is
    (1) rational (3) imaginary
    (2) irrational (4) nonexistent

9.5 Exponential Equations

Just as an exponential function is a function in which the variable is in the exponent, an **exponential equation** is an equation in which the variable appears in an exponent. Let us look at an example.

Solve for x.

$$3^x = 9$$

We know that $9 = 3^2$, so we know that x must be equal to 2. What if we had to solve the equation $3^{x-1} = 9$? We know that $9 = 3^2$, so $x - 1 = 2$ and $x = 3$. What if we had $3^{2x-3} = 9$? Again, since $9 = 3^2$, we have $2x - 3 = 2$ and $x = \frac{5}{2}$.

Think about how we are solving these equations. We are rewriting both sides of the equations with the same base, setting the exponents equal to each other, and solving the resulting equation.

Remember: If $b^x = b^y$, when $b \neq 0, 1$, then $x = y$.